Remarks on Wittgenstein's Philosophy: 
Private Language and Meaning

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ABSTRACT. This essay is a critical analysis of some themes in Wittgenstein's later philosophy. It is not primarily Wittgenstein-exegesis. Much more modestly, my purpose is to express my own thoughts about some questions which Wittgenstein has treated in his writings. It is the first in a series of two articles. The second article, "Remarks on Wittgenstein's Philosophy: Philosophical Method and Contradictions", will occur in Volume 43, 2008, of the present YEARBOOK.

Section 1, "The Private Language Argument". An independent argument is given for Wittgenstein's thesis that there is no private language. I show that psychological terms in ordinary language, in contrast to an implication of Wittgenstein's own private language argument, in their meanings do contain references to inner states, processes, or events.

Section 2, "Meaning". Wittgenstein's definition of meaning as use in the language is criticised. Meaning is instead identified with something in the content of a conscious mind. Applications are given to some suggestions in philosophy of language by Chomsky, Harman and Fodor, Grice, and Kripke. For orientation, I also include here the abstract for the second article, "Remarks on Wittgenstein's Philosophy: Philosophical Method and Contradictions".

Section 1, "Philosophical Method". Wittgenstein's conception of philosophy as language therapy is criticised. Instead philosophy is construed as foundational research. Wittgenstein's statement that mathematical logic cannot contribute to progress in philosophy is repudiated. Several examples of ideas and results in mathematical logic which have led to the solution of philosophical problems are given.

Section 2, "Contradictions: The Wittgenstein-Turing Debate". In lectures on the foundations of mathematics given in 1939, Wittgenstein claimed that contradictions in mathematical theories are harmless. A debate ensued on this question between him and Alan Turing. In support of Turing's standpoint, I use the theorem on Taylor series, Church's Theorem, and Gentzen's Cut-Elimination Theorem to show that Wittgenstein's standpoint is untenable.
1. The Private Language Argument

1.1 Anthropology. I assume that a human being at any time is a well-defined system. He consists of a body and has a central nervous system, including a brain. I assume that a human being has the capacity of self-awareness (or self-consciousness). The body and the central nervous system can be studied from the outside, for instance by the methods of physiology, psychology, neurology, and molecular biology. The self-awareness implies that the individual, at least to some extent, is aware of himself as a system of limited spatial extension, of his body and its parts and of their function. I also assume that self-awareness implies a direct consciousness in the individual of some of the states, processes, and events in his own central nervous system. Thus the body and the central nervous system, and in particular the brain, can also be studied from the inside, via introspection. As a consequence we can know a lot about our own brain independently of whether we know that we have a brain and which biological properties it has. Self-awareness even implies an understanding of the fact that other individuals of the same species, Homo Sapiens, have similar bodies and minds as one self and are self-aware and self-conscious.

1.2 REMARK. Some of Wittgenstein's statements in his later philosophy have been interpreted to mean that he did not agree with the claim that self-awareness implies a direct consciousness in the individual of states, processes, and events in his own central nervous system. Instead he might have had a behaviourist view to the effect that we only have indirect knowledge of what is going on in our brains and central nervous systems via observation of our own behaviour and tendency to behaviour. Whatever may have been Wittgenstein's view, I myself believe in our direct awareness of our central nervous systems (and will use it freely in the present essay) for the following reasons.
(1) Sometimes we dream during sleep and remember the dream when we wake up. We remember the dream as a vivid visual observation of a scene. A similar vivid fantasy could not be constructed by the brain from observations of our own behaviour immediately after waking up. This shows that we have been directly aware of the inner activity going on during the dreaming. Thus a direct awareness of an inner process is, indeed, possible and can be assumed to be present in the case of other mental states and processes as well.

(2) Self-awareness has been shown to be present in the following species apart from Homo Sapiens: the primates (chimpanzees, bonobos, orangutans, and gorillas), dolphins, and elephants. Self-awareness, including awareness of states etc. in ones own body and brain, is clearly a very useful faculty in the struggle for life. If such an awareness is direct, it can be attained in a fragment of a second. If the awareness were indirect, it should be much slower and much less useful in critical situations than it actually is.

1.3 DEFINITION. In *Philosophische Untersuchungen*, Wittgenstein investigates the idea of a private language in §§243-309, and with a definition of the concept in §269. A *private language* is a language which, even in principle, is impossible to understand by anybody except a single user U of the language.

1.4 First Thesis. Wittgenstein asks whether there is any private language. His first claim is:

(4-1) *There is not and cannot be any private language.*

I agree with this thesis and now give my own argument for it.

1.5 The Church-Turing Thesis. (1) An arithmetical function is computable iff it is recursive.
(2) Every algorithm, whether numerical or non-numerical, can be represented by a recursive function.

I assume the Church-Turing thesis and use it freely in the sequel.

1.6 ANALYSIS. The following seems to be a reasonable characterisation of some essential features of all languages.

A language \( L = (E, F) \) consists of a set \( E \) containing all the expressions of the language (for instance, words, phrases, and sentences) and a semantic function \( F \) which to every expression in \( E \) assigns a meaning. If some of the expressions are homonyms, \( F \) must be multiple-valued. A user \( U \) must be able to recognise the expressions in \( E \) and distinguish them from other objects not belonging to \( E \). This implies that the characteristic function \( K_E \) of \( E \) must be recursive. To understand the language \( L \), \( U \) must have and master a complete definition of the semantic function \( F \). This implies that \( F \) must be determined by a finite rule which can be and is stored in \( U \)'s memory and which can be followed by \( U \) in the calculation of \( F \). Therefore, according to the Church-Turing thesis, \( F \) must be a recursive function. (It can be argued that \( K_E \) and \( F \) must be primitive recursive functions; but we do not need this fact in the present essay.)

1.7 ARGUMENT. I give my own argument for the thesis 1.4 on the non-existence of a private language. Suppose that the user \( U \) masters the language \( L = (E, F) \). I now show that it is possible, at least in principle, for another individual \( R \) to understand \( L \). As part of the definition of the problem assigned to \( R \), he is given the syntactic part of the language, that is, he is informed about the characteristic function \( K_E \). His task is to try to figure out a definition of the semantic function \( F \). We note that since \( U \) masters the semantic function \( F \), \( U \) has a complete definition of \( F \) as a recursive function in his memory. \( R \) can study \( U \)'s brain from the outside and he can, in principle, obtain sufficiently much in-
formation about the content of U's memory so that he has a complete definition of F. The memory of a computer can be scanned from the outside and the information in the memory examined. We are not forced to use the computer's own scanning function. I see no reason why it should not similarly in principle be possible to scan the memory of a human being from the outside and examine his or her knowledge. The fact that it is knowledge about meaning makes no difference. What is so special about knowledge of meaning? Equipped with F, also R will be able to understand L which is then not private. (Using a similar way of reasoning, one can refute Wittgenstein's dictum that if a lion could speak, we should not be able to understand the lion.)

1.8 REMARK. Let L be a language fragment used by U. The most common method used for another person V to reach an understanding of L is via an interpretation I. U can to some extent via the self-awareness look into his own central nervous system, including how the expressions of L are associated with meaning. V has no such privileged access. For him, U's mind is a black box. V's interpretation consists in formulating a theory about the black box. It is a theory about how expressions in L are connected with meaning. The formulation of theories about black boxes is a well-known type of problems in the sciences. Thus for instance the atomic nucleus can be considered a black box and nuclear physics is the theory about the inner structure of this black box. Characteristic of black box theories is that they cannot be verified; but they can in principle be falsified, and this suffices to make the enterprise meaningful and scientific. This is also how theologians, philologists, and historians of philosophy work: formulate hypotheses of interpretation and try to falsify them. Wittgenstein demands (§258 in Philosophische Untersuchungen) that an interpretation must involve a criterion by which the association of symbol with meaning can be shown to be right or wrong. I suppose that the criterion must be possible to apply to decide whether the association is correct, that is, whether the expressions of L are used
correctly. Then the criterion is an algorithm. This implies that an interpretation must be verifiable; but this is too strong a demand on a black box theory, both in physics and in semantics. Falsifiability suffices.

1.9 REMARK. Wittgenstein's own argumentation for Thesis 1.4 is very different. There is some uncertainty about what he meant by the private language argument. In this section, I will proceed by taking his statements and examples for what I consider to be their face value. In Section 2, I will briefly consider another, weaker interpretation. When Wittgenstein draws conclusions from the private language argument, he seemingly uses the stronger interpretation considered in the present section. Therefore it will be our main alternative. In the argument, he takes what he apparently considers to be a proposal for a private language: the fragment of language concerned with pain and other sensations. He then tries to show that such expressions do not in their meanings contain components which refer to something inner and private (inner and private in the mental sense). Thereby he assumes that if an expression belonging to psychological language, for instance an expression concerned with a sensation, has a reference to something inner and private, it cannot be understood by anybody else than the user U and therefore is a private language. This assumption is not trivial; and it will turn out actually to be false. It turns out that psychological terms do refer to inner states, processes, and events in the mind, and they do it in the same way that physical terms refer to states, processes, and events in the external, physical world.

1.10 Second Thesis. Thus Wittgenstein has a second thesis:

(10-1) Expressions belonging to philosophical psychology, for instance expressions concerned with sensations, do not in their meanings contain a reference to inner and private states, processes, or events.
1.11 QUOTATION. "How are words related to sensations? […] For instance the word 'pain'. The following is a possibility: Words are connected with the original, natural expression of the sensation and replace it. A child has hurt himself, he cries; then the grown-ups talk to him and teach him exclamations and later sentences. They teach the child a new pain behaviour. […] the verbal expression of pain replaces the crying and does not describe it." (PhU, §244)

(* The quotations in sections 1 and 2 are my own translations into English from Philosophische Untersuchungen.*)

1.12 REMARK. Thus Wittgenstein claims that expressions like "I have pain", "I have a headache", and "It hurts in my left knee when I walk" really are only a kind of pain behaviour on a par with crying or groaning. It does happen incidentally that such expressions are used the way Wittgenstein claims. Thus it has happened that I, as a kind of groaning, have said "It hurts, oh, it hurts so much" at occasions when I have hurt myself against something. But it is hard to believe that his claim is generally true. For example:

(1) The child already has a natural register of pain behaviour consisting in crying and groaning. What is the idea of expanding the register?

(2) Crying and groaning are generally quite inarticulate. Many uses of expressions for pain and other sensations are in contrast very detailed and specific. Examples occur when we see a doctor and try to give as precise a description of our symptoms as possible. There a patient might talk to the physician as follows: "I have a strong headache. It is like a heavy pressure over the forehead. It is particularly intense in the morning. It decreases somewhat during the day and is slightly better in the evening. Still the pain makes it difficult for me to fall asleep in the evening, and I usually wake up two or three times during the night because of the pain. It is getting a little worse every day. The headache started c. six weeks ago; and during the last 2-3 weeks, it has been a real ordeal I dare
say." The language used is descriptive rather than expressive. The patient's attitude is one of observation and recollection. The procedure is much more similar to the description of something in space and time than it is to groaning. It is as if he is describing an inner state.

For this reason I find the following to be a more reasonable picture.

\[(12-1) \quad \text{Expressions in the language used for pain, feelings, sensations, perceptions, fantasies, dreams, thoughts, ideas etc. refer to inner states, processes, or events, and this reference is an essential component in the meaning of the expressions.}\]

Thus I adhere to this unsophisticated common sense view as opposed to Wittgenstein's sophisticated philosophical view. I now review and analyse some of Wittgenstein's arguments against Thesis (12-1) and also some arguments against the same thesis and for Thesis (10-1) advanced by other philosophers. During the process, my own reasons for believing in Thesis (12-1) will become clear.

1.13 QUOTATION. "To what extent are my sensations private? - Well, only I can know whether I really have pain; another person can only presume it. - This is in one way false and in another way nonsense. When we use the word 'know' as it is normally used (and in what other way should we else use it!), then other people very often know when I have pain." (§246)

1.14 REMARK. My sensations are private to the extent in which I have access to them and other people do not. Via the self-awareness, I often know immediately about my own pain while other people do not. In principle, it is possible for other people by studying my body and central nervous system from the outside to know everything about my pain. But under the circumstances which normally characterise human social life, most of the methods and equipment needed for such investigations are not available. In particular, they are not pre-
sent in the circumstances under which we acquire a language. Every normally developed person has via the self-awareness a privileged access to immediate knowledge about states and processes in his own central nervous system. It cannot apriori be excluded that this privileged access plays a role in the way we learn a language and the meaning we assign to linguistic symbols.

1.15 QUOTATION. "Of myself, it is not at all possible that I say (except possibly as a joke) that I know that I have pain. What is that supposed to mean - except possibly that I have pain." (§246)

1.16 REMARK. The linguistic usage of the word "pain" is not as unambiguous as Wittgenstein claims. In one usage, I can have pain without knowing it. In another usage, having pain implies knowing that one has pain so that they are one and the same thing.

(1) "I know that I have pain" can mean that I am conscious of my pain. Via the self-awareness, it is possible to be directly conscious of a pain. But it is also possible at times not to know of a pain though it is there, for instance because my attention at that moment is totally occupied with something else or because the self-awareness is absent. Thus a person can have pain when he is sleeping and not know about it because his self-awareness during sleep is turned off. The pain can be inferred from his behaviour during the sleep by people observing him, for instance from his groaning while sleeping. In this sense of "pain", pain is a state $\Pi$ in the brain of which one may be or not be aware.

(2) There are also people who are only willing to ascribe pain to a person who is aware of the pain. For them, a person can have no pain during the sleep no matter how much he groans and shows other pain behaviour. In this sense of "pain", pain is the kind of state $\Sigma$ in the brain which consists in a direct awareness of a state of type $\Pi$. 
I am interested in showing that words like "pain" can - and do - refer to inner states. Wittgenstein's argument in Quotation 1.15, if it is meant as an argument, does not help to exclude this possibility. Because in the first sense of "pain", the word refers to an inner state of type $\Pi$. In its second sense, "pain" refers to an inner state of type $\Sigma$.

1.17 QUOTATION. "Let us imagine the following case. I want to keep a journal of how a certain sensation recurs. For this purpose I associate it with the symbol 'S' and write this symbol in a calendar every day when I have the sensation. - I first want to call attention to the fact that no definition of the symbol can be formulated. - But I can still give it to myself as a kind of ostensive definition! - How? Can I point to the sensation?" (§258)

1.18 REMARK. It is a misunderstanding to believe that pointing is a necessary part of an ostensive definition. When we, in order to define a phenomenon to a person P by an ostensive definition, point to the pertinent instance of the phenomenon, the sole function of the pointing is to draw P's attention to the instance. If we are certain that P at the moment already gives sufficient attention to the phenomenon, we can give the ostensive definition without pointing first. Similarly in Wittgenstein's example, if my attention is already focused on the sensation, or the memory of the sensation - and whether this is the case or not, I can decide via the self-awareness - then an ostensive definition can be given without any previous pointing.

The way we teach children words for different kinds of pain follows this pattern. We wait until the child has become a victim of some kind of pain and we are certain that the pain fills his or her attention. Then we comfort the child by, among other things, speaking to him or her and using words for the relevant kind of pain. I was myself a witness to the following episode. One evening, I
visited a couple who had a little son, Johannes, two years old. A candle was lighted on the table. Johannes got curious about the fire and climbed up on the table via a chair. To examine the light, he tried to stick his right index finger into the flame and got burned. He started immediately to cry and the tears flowed. The parents were quickly in place and comforted him and spoke to him: "Have you hurt yourself, Johannes? Oh, you burned your finger." The child learned quickly and repeated through tears: "Yes, I hurt myself! I burned my finger!"

1.19 QUOTATION. "How? Can I point to the sensation? - Not in the usual sense. But I pronounce or write down the symbol and at the same time focus my attention on the sensation - I so to speak point do it in my mind. - But what is this ceremony good for? Because it appears to be nothing else! The purpose of a definition is to determine the meaning of a symbol. - Well, it is done precisely by my focusing my attention; because by doing that, I impress the connection between the symbol and the sensation in my mind." (§258)

1.20 REMARK. There is no doubt that an association between a new feeling and a symbol can be established in just the way proposed by Wittgenstein's fictive partner of discussion and denied by Wittgenstein. When I was between 18 and 22 years old, I had at four occasions a strange feeling which I have otherwise not had, neither before nor after. When the feeling came, I was sitting at a table alone in a room and for a couple of minutes, a feeling of inner peace fell on my mind at the same time as the table and everything else in the room and in the world was felt to be far away. In this state, I asked myself, as an alarming and existentially important problem and not only as an academic question: "Why is there something and not rather nothing?" I called the sensation "the remoteness feeling". In my mind, I connected this expression with the feeling in a kind of ostensive definition in just the way which Wittgenstein claims cannot be done. The expression got its meaning by the "ceremony". This worked perfectly well.
The ceremony was good for something. For instance, I can say that I have not had the remoteness feeling since I was twenty-two, and altogether I have had it four times. Since what Wittgenstein claims to be impossible actually has been done, something is wrong about his opinion that expressions for sensations cannot refer to inner states. The further analysis must show what is wrong.

I consider a couple of objections to this example which I have encountered. One says that an expression in a private language is not allowed to contain any words from ordinary language. I am not convinced of the validity of this objection. Anyhow the example should work just as well if I had called the sensation "abracadabra" or "S". Another objection claims that one can give new names to inner sensations, but they do not belong to a language. The reason for this claim seems to be that such new expressions cannot be used to communicate with other people. This objection is not valid. A language can be used for several purposes other than communicating with other people. Some people write a diary only to support their own memory without ever intending anybody else than themselves to read it. Some people even write a diary without intending themselves to read it again. They write it just as a means for the expression of their own thoughts and in that way clarify them. Language can not only be used for interpersonal communication. Language can also be used for intrapersonal communication.

1.21 QUOTATION. "Well, it is done precisely by my focusing my attention; because by doing that, I impress the connection between the symbol and the sensation in my mind. - 'I impress it in my mind' can, however, only mean: this procedure brings about that I in the future remember the connection correctly. But in our case, I have no criterion of the correctness. Here one might say: correct is that which always appears correct to me. And that only means that here it is not possible to talk about 'correct'." (§258)
1.22 REMARK. The symbol "S" refers to a *type* of sensation. Therefore an understanding of the meaning of "S" must include a method for distinguishing such individual sensations which are of type S from those which are not. The method is a recursive decision algorithm associated with S. Such algorithms use to contain components which work automatically and therefore are unconscious. For a predicate like S, the decision method is the essential component in the meaning. Wittgenstein's demand is then that there must be a criterion for the correct association of "S" with the decision method. Since only I know about the connection, the criterion can only be the following: the information about the algorithm and its connection with "S" is stored in my memory at the occasion of the ostensive definition and after that, the information is unchanged and accessible to me, and the unconscious components of the algorithm work as required. As long as the criterion is satisfied *as a matter of fact*, this suffices for practical purposes, that is, for my own private use of the sign "S" in a correct way. Of course, it does not suffice for communicating with other persons about S; but since I write the diary about S only for my own private purposes, this does not matter. Wittgenstein's demand could be that a criterion of meaning must be intersubjective. Such a demand is justified in the case of an expression used for communication between individuals; it is anything but obvious if the expression is used only for private purposes of one person. Justified or not, the demand can in principle be satisfied even in the latter case. By examining the brain from outside, it is in principle possible for somebody else to verify or falsify that the information about the algorithm and its connection with "S" is unchanged in my memory and accessible to my mind whenever I want it and that the automatic components of the algorithm work properly.

1.23 QUOTATION. "What reason do we have for calling 'S' the sign for a *sensation*? 'Sensation' is after all a word in our common language and not in a lan-
guage intelligible only to me. The use of this word is in need of a justification which everybody will understand." (§261)

1.24 REMARK. There is a decision method for S, and there is a decision method for sensations in general. If the decision algorithm for sensations gives a positive output for a given input whenever the decision algorithm for S gives a positive output for the same input, then it is justified to call S a sensation. The situation is the same here as with many other pairs of predicates, including such predicates which are concerned only with phenomena in space and time.

1.25 ANALYSIS. It is useful also to analyse how words connected with perception get meaning. Suppose I want to teach a child U the meaning of the colour words "red", "green" etc. I take care that the appropriate external physical circumstances are present: objects of various colours are placed in a visible place in front of U and at a suitable distance; it is sufficiently bright and the light falling on the objects is white; there is nothing opaque between U's eyes and the objects, etc. I place a sequence of objects of different shades of red before the child and say for each object "This is a red object." I continue with a sequence of green objects of different nuances. After a while, U has understood the ostensive definitions and can himself with great certainty select what is red, what is green, and what is neither. U has understood the meaning of "red" and "green". This implies that he has associated one algorithm with the word "red" and another algorithm with the word "green". It is not important whether U's algorithms are the same as mine. What does matter is that they classify the same objects as red and the same objects as green as my algorithms do.

What is the criterion of redness in an algorithm used by human beings unaided by any apparatus? (Note that this is another sort of criteria than those wanted by Wittgenstein in §258.) It is fairly easy to build a robot which identifies the colour of an object. It measures the frequency of the light reflected from the surface
of the object. If the bulk of the light beams have frequencies in the interval from $4.3 \times 10^{14}$ Hz to $4.8 \times 10^{14}$ Hz, then the object is classified by the robot as red. There are similar criteria for the other colours. The algorithm and the criteria used by a human being unaided by instruments are different. In particular, our criteria are not numerical. We distinguish colours by the phenomenological quality of the impression which the watching of a coloured object leaves in the mind. Red has one phenomenological character, green another. Such criteria are made possible by the self-awareness.

The kind of activity which U and I carry out in order that U should learn the meaning of the colour words is called a language game by Wittgenstein. I now try to analyse precisely what happens during such a language game. The algorithm associated with the colour word "red" gives the basic meaning of the word. Since I attach a meaning to the word "red", I have one such algorithm associated with "red". This includes the criterion for redness based on the phenomenological quality of the impression which a red object leaves in my mind. This reference to an inner, private state in me is part of the meaning I attach to "red"; but since it is inner, I cannot convey this part of the meaning to U during the language game. The part of the meaning of "red" which I can transfer to U is the extension of redness: which objects are red and which are not. U has understood the meaning of "red" when he has found or developed an algorithm which selects precisely the same objects as red as my algorithm does. U's algorithm also contains a criterion of redness based on the phenomenological qualities of red in his mind. His algorithm need not be the same as mine, and the phenomenological character of redness in his mind need not be the same as in mine. What is important is that the two algorithms give the same results on redness and non-redness. U's criterion of redness is part of the meaning which he attaches to "red". Thus also the meaning U attaches to "red" contains a reference to something internal and private in U's mind. Still U and I, and everybody else who use
"red" in the same way as we do, can communicate on redness, and understand each other. (Incidentally, this shows that meaning and use are not the same.)

Does a person with an inverted colour spectrum associate another meaning with the word "red" than a person with a normal colour spectrum does? Yes! They have different criteria of redness. The criterion of redness is part of the meaning of "red". Therefore the meanings they attach to "red" must differ. This is no obstacle to communication between them. What matters in the interpersonal communication is that they use the word "red" in the same way, that is, that they classify the same objects as red with their different criteria and different meanings attached to "red".

It is easy to see that the meanings of words for pain and other sensations, in essentially the same way as colour words, contain a reference to inner, private states. Similarly, words for the other psychological phenomena listed in Thesis (12-1) contain references to inner states, processes, and events.

1.26 EXAMPLE. (I) We can make aesthetic judgements about colours: "Green is beautiful." "Red is pleasant. I enjoy red." "Red and green match each other well." We consider the example:

(26-1) Red is pleasant. I enjoy red.

Such a judgement presupposes self-awareness. By saying so, I express how I react to the phenomenological character of red. The statement is intelligible to anybody who knows that I am self-aware and that the meaning I attach to "red" contains a reference to the phenomenological character that red has to me (whatever it may be). The semantic ideas in Analysis 1.25 make aesthetic judgements intelligible.

(II) A dream is a mental process occurring during sleep and which produces images deceptively similar to the images produced by watching a real scenery. Children learn quite early on in their lives to distinguish dreams from percep-
tions. Dreams are like perceptions except that they are not caused by any external scenery. Therefore both "dreaming" and "perceiving" have in their meanings a reference to the existence of an inner process. "Dream (content)" and "perception (content)" have in their meanings a reference to the existence of inner images.

1.27 EXAMPLE. Suppose a person P is colour-blind. This is a handicap, not least in the traffic. To reduce his handicap, P buys an optical instrument which can discern colours. The device works, as described in Analysis 1.25, by measuring wave frequencies and using numerical criteria for the colours. When an analysis has been made, the instrument gives the colour of a surface it has been exposed to via a display where the user of the instrument can read the colour of the surface scrutinised: 'red', 'green', or whatever it is. Using the device, P can classify objects according to colour with about the same certainty as a person with full colour seeing who uses only his unaided eye and the phenomenological criteria. Still P will not be able to understand aesthetic judgements about colours. This shows that the hypothesis about the existence of an inner image of, for instance, red in other people and the existence of phenomenological qualities in such an image is part of the full meaning of the word "red". Such an existence hypothesis is an anthropological hypothesis. Thus an empirical, anthropological hypothesis can be part of the fully developed meaning of a colour word. It also shows how a reference to something inner can be part of the meaning of a colour word. This part of the meaning does not affect all uses of colour words, for instance not when we classify objects according to colour; but it does affect the use of colour words in aesthetic judgements.

1.28 QUOTATION. "If I say about myself that I only know from my own case what the word 'pain' means, - don't I have to say the same about other people too? And how can I then generalise this one single case in such an irresponsible
manner? Nevertheless everybody says about himself that he only knows from himself what pain is!" (§293)

1.29 REMARK. Analysis 1.25 shows that I only know from myself the full meaning of the word "pain", in agreement with the common view; and it is the same with other people. The generalisation is, however, not irresponsible. It is based on the very reasonable and well-founded empirical hypothesis that human beings are composed and function in mainly the same way (as I do). Analysis 1.25 shows that all we need to assume is that other people are self-aware and decide the extensions of concepts for colours and sensations by algorithms which are built into the organism itself and have a criterion for the output which is immediately accessible to the person's consciousness.

In my philosophy, the common sense opinion, that everybody only knows from himself what pain is, is correct. Analysis 1.25 shows that the meaning of "red" has two dimensions: the extension of redness and the inner, phenomenological character of redness. Similarly, the meaning of "pain" has two dimensions: the extension of the predicate "pain" and the inner, phenomenological character of pain. The extension of "pain" can be decided by external, for instance medical, criteria. But only a person who has himself suffered pain can understand the full meaning of the word "pain". Most of us experience only quite easy pain in life. Some forms of cancer are known to give very intense and permanent pain which places an enormous physical and psychological strain on the patient. A person who has gone trough such an ordeal can truly say that this experience has given the meaning of the word "pain" a new dimension.

1.30 QUOTATION. "Suppose that everyone had a box in which there were something called 'a beetle'. No one is ever able to look into another person's box; and everybody says that he knows only from the sight of his own beetle what a beetle is. - Then it might be possible that everybody had a different thing than
the others. In fact, one might even imagine that such a thing is in constant change. - But if, in spite of that, these people still had a use for the word 'beetle'? - In that case, it would not be a designation for a thing. The thing in the box does not at all belong to the language game, not even as a something: because the box might even be empty. - Indeed, this thing in the box can be 'reduced'; it disappears from the picture whatever its nature may have been.

In other words: If we construct the grammar of the expression for a sensation according to the model 'object and name', then the object slips out of consideration as irrelevant." (§293)

1.31 REMARK. Refer to Analysis 1.25 and think of the boxes as U's mind and my mind and the beetles as the image of red in U's mind and the image of red in my mind. Then it is, indeed, possible that the two images are different. As a matter of fact, it is probable that they are not completely similar. It is even possible that the images change over time. Also this is actually quite probable because there are good reasons to believe they are not exactly the same over a whole lifetime. This is of no importance as long as the algorithms, in which the images have a role, distinguish in the same way between red and non-red objects. Analysis 1.25 shows that the character of the image of red has no role in the language game for "red" played by U and me, just as claimed by Wittgenstein. But he is wrong in concluding that then it has no place in the meaning of "red" either. In contrast to what Wittgenstein erroneously believed, the meaning of words like "red", "green", "pain" and "sensation" is not completely determined by the language game as the example in Analysis 1.25 clearly shows. The "beetle in the box", that is, the reference to the inner state, also contributes to the meaning as explained in Remark 1.29. Therefore "the thing in the box cannot be reduced." Wittgenstein is wrong in his claim that "the box might also be empty." For a predicate like "red", U must have an algorithm for the extension of redness; and U's image of redness is part of his algorithm. That image is in Witt-
genstein's analogy "the beetle in the box". The same obtains for predicates like "pain" and "sensation".

1.32 REMARK. Other philosophers, for instance Michael Dummett, have given support to Thesis (10-1): Everything that affects the meaning of expressions in a public language like English must be something which is itself public and knowable by anyone, they claim. In particular, the meaning of such expressions cannot contain any reference to private mental states, processes, and events of the sort discussed above. I briefly comment on two arguments from this post-Wittgensteinian literature.

1.33 The Acquisition Argument. Suppose L is a public language containing an expression "E" whose meaning depends on the nature of certain private states in competent speakers. Let U be a person learning the language L and in particular acquiring an understanding of the expression "E". Then U must come to use it in connection with his own private states of the same kind as those associated with "E" by the competent speakers. But since these states (in the competent speakers) are private, they are not accessible to U, so how can U identify the right state in himself to associate with "E"?

1.34 REMARK. The argument rests on the assumption that meaning is use. If the meaning of "E" is identical with the use of "E", then the problem pointed out in the acquisition argument does arise. But as mentioned in Analysis 1.25, meaning cannot be identified with use. It is useful to consider two examples.

(I) In Analysis 1.25, we considered an example where I teach U the meaning of the colour words. The meaning of the predicate "red" for me is the algorithm I associate with the word. I teach U the meaning of "red" by helping him to find or develop a suitable algorithm to associate with the word "red". It need not be the same algorithm as mine and, in particular, the output state in U's mind of the
algorithm, the colour image, need not be the same as mine. What is required is that the two algorithms assign the same extension to "red". When this is satisfied, U and I use the word "red" in the same way. We see that the meaning of the word "red" has two components. One component is the extension. It is public and can be identified with the use. It is the same for all users. The other component is private. It consists of the algorithm for the extension of "red" and notably the inner state which in the mind is conceived of as the image of redness. It may vary from individual to individual as long as it gives the same output to the question about what is red and what is not. The question in the acquisition argument is: How can U identify the right state in himself to associate with "red"? The answer is that there is no the right state. U must just find some algorithm and some state which assigns the same extension to "red" as my algorithm and state do and as any other competent speaker's algorithm and state do.

(II) The second example is the aesthetic judgement "Red is pleasant. I enjoy red" from Example 1.26. It presupposes self-awareness in the speaker. The speaker V expresses how he reacts to the phenomenological qualities of his inner image of red. For a listener U to understand the statement, U must assume that there exists an inner image of red in V and that the image has some phenomenological qualities. These existence conditions, he can infer from his belief that V, and most other human beings, are quite similar to himself. For the understanding, he needs, however, know nothing about just what the image is and what phenomenological qualities it has. Suppose U learns from V and also wants to make an aesthetic statement, for instance: "I must admit that I am not very fond of red. It makes me sad." Here there is a right state for U to identify, namely the one he learned from me to associate with the word "red". But it need not be the same state that V has associated with the word "red". U's judgement on how he reacts to the phenomenological qualities of red must be based on just that state which he himself has associated with "red" when he learned the meaning of "red". All V needs in order to understand U's judgement is again the existence condition
that there exists an inner image of red in U and that the image has some phenomenological qualities.

1.35 The Manifestation Argument. This argument views the process from the teacher's side. How can the teacher know that the learner U has associated the right meaning with the expression "E"? To grasp the right meaning is to connect a particular private state rather than others with "E". Because the states in U are private, it is impossible for the teacher to check that U has associated the right inner state with "E" and it is impossible for U to manifest the association.

1.36 REMARK. To understand the meaning of an expression is not to associate one particular meaning with the expression, the same meaning as the teacher. It is to associate some meaning with the expression, a meaning which determines the same use of the expression as the teacher's use. Again the example in Analysis 1.25 is useful.

When U learns the colours from me, the meaning eventually attached to "red" by U is determined by the algorithm, including his image of red, which he uses to determine the extension of the predicate "red". The meaning I associate with "red" is determined by my extension algorithm and my image of red. U's algorithm and U's image of red may well be different from mine. Then the meaning U associates with "red" is different from the meaning I associate with "red". For U to have understood the meaning of "red", his algorithm must yield the same result on which objects are red and which are not as my algorithm does. This can easily be checked by the teacher. When this is satisfied, U and I use, in the given context, the word "red" in the same way. This is perfectly compatible with the possibility that the private components of the meanings which U and I assign to "red" are different.
1.37 CONCLUSION. Wittgenstein's first thesis states that there is no private language, that is, there is no language which can only be understood by its single user but which cannot, even in principle, be understood by anybody else. The thesis is correct; but Wittgenstein's argument for it is erroneous.

Wittgenstein's second thesis states that expressions concerned with sensations do not in their meanings contain a reference to inner, private states. This thesis is false. The arguments advanced by Wittgenstein and Dummett are insufficient and erroneous. The meaning of a colour word can be identified with the criterion by which we distinguish that colour from other colours and other phenomena, namely the phenomenological qualities of that colour in the mind. Similarly the meaning of a word for a sensation can be identified with the criterion - based on the inner, phenomenological qualities - by which we distinguish that sensation from other sensations and other phenomena. The exact character of these phenomenological qualities plays no role in the use of the words; but in aesthetic judgements, the existence of the phenomenological qualities is essential.

1.38 REMARK. There really are two interpretations of Wittgenstein's private language argument.

In the weak interpretation, the purpose of the argument is to disprove dualistic theories of the mind. The idea seems to be that if the mind were made of some other kind of stuff than the body and the meaning of words were located in the mind, then we should not be able to understand each other because no hypothesis about what a person meant by a word could ever be verified since we, according to this kind of theory of the mind, cannot have access to other people's minds. This weak form of the argument is wrong because a dualistic theory of the mind makes the mind a black box. A theory about the inner of a black box need not be and cannot be verifiable. It suffices that it is in principle falsifiable. An hypothesis of what state in the immaterial mind is associated with a word for
pain could be in principle falsifiable. Therefore a dualistic theory of the mind does not have as a consequence that psychological language must be private.

There is also a strong interpretation of the private language argument. According to this interpretation, terms in the psychological language cannot refer to something inner even when we have a monistic theory of the mind. This stronger interpretation is the one used above in the present section. The reason for this choice is that when Wittgenstein draws conclusions from the private language, for instance in his theory of meaning considered in the next section, he clearly uses the strong version of the argument. As shown above, if we identify the mind with the brain as seen from inside the brain itself, then a person's statements about states, processes, and events in his own mind could actually be verifiable given that we have sufficiently technologically advanced and sensitive instruments at our disposal. It might be objected that this is of no relevance because in ordinary situations of language use and language learning, we do not have, and maybe cannot have, such technological means at our disposal. Given this, the mind becomes a black box. Just as in the weak interpretation, an hypothesis about the meaning of a word consisting in associating the word with an inner state in the mind (brain) of the speaker is in principle falsifiable. That is all which is needed for a theory of the meaning of psychological terms consisting in associating the meaning of such terms with inner states. How can such an hypothesis be falsified? It is false if it has consequences which are incompatible with the speaker's behaviour, including his or her linguistic behaviour. Thus the outer behaviour is the testing stone for the inner state.

2. Meaning
2.1 INTRODUCTION. Maybe the best known statement in the philosophy of the later Wittgenstein is:

\[(1-1) \quad \text{Meaning is essentially use.}\]

This is Wittgenstein's meaning theory. The following quotation gives another formulation of the idea: "For a large class of cases - though not for all - in which we employ the word 'meaning', it can be defined thus: the meaning of a word is its use in the language." (§43)

\[(1-2) \quad \text{The meaning of a word is its use in the language.}\]

2.2 REMARK. A question to consider is the interpretation of the statements. First: What does "essentially" mean here? I will take it to mean that \textit{fundamentally} and \textit{primarily}, meaning is use (in the language). Occasionally one hears or reads uses of words which do not agree with their ordinary use in the language. To the extent in which such uses are intelligible, they are secondary and derived. For example, when Franz Kafka was dying in cancer and suffered severe pain, he is reported to have said to his friend Max Brod: "If you don't kill me, then you are a murderer!" Here Kafka uses the word "murderer" in a way in which it is not used in ordinary language. Nevertheless the statement is completely intelligible. It is a metaphorical way of saying: "If you don't relieve me of this terrible pain (which can only be done by killing me), then you commit a (moral) crime at the level of murder." It shows that Kafka has used "murderer" in a secondary sense which is parasitic on its common and primary meaning.

Second: What can Wittgenstein's motives for identifying meaning with use be? The answer is implicit in the private language argument. This argument is meant to support Wittgenstein's thesis that no word in the public language can in its meaning have a reference to an inner state or process. Therefore the word "meaning", in particular, does not in its meaning have a reference to something which is psychologically inward. Then only the public and outward \textit{use} of a
word is left to identify its meaning with: Meaning is use. Thus Wittgenstein's theory of meaning is really just a corollary to the private language argument.

One consequence of Wittgenstein's definition of meaning is that the meaning of a word or expression cannot exist independently of and before the word or expression. Formulated as a slogan (my own formulation, not Wittgenstein's):

(2-1)  \textit{No meaning without language.}

In the present section, I will argue the following theses in opposition to Wittgenstein's views:

(2-2)  \textit{Meaning can be identified with something that is on a self-conscious individual's mind. Meaning determines use but cannot be identified with use.}

(2-3)  \textit{Language is an instrument for the expression of meanings.}

(2-4)  \textit{Meaning precedes ontologically and temporarily language. Thus meaning can exist prior to and independently of language.}

Again these ideas coincide with the primitive common sense views of the man in the street as opposed to Wittgenstein's sophisticated philosophical standpoint.

2.3 ANALYSIS. Since Wittgenstein's theory of meaning is a corollary to the private language argument and this argument is invalid, it might be possible that even the theory of meaning is invalid. Since words for colours and sensations in their meanings do contain a reference to a psychologically inner state, it might be that even the word "meaning" does so. It might even be that meaning can be identified with something inner.

It is useful to consider how the word "red" gets its meaning. The eyes and the central nervous system contain a mechanism for seeing. Most of the mechanism works without its possessor being conscious of how it works. A mechanism is essentially the same as an algorithm. One part of the algorithm may become
conscious, namely the final result of the working of the algorithm: the image. We can discern three aspects of the attribution of meaning to the word "red".

(1) The phenomenological qualities of the image of red. They are the criterion by which we distinguish red from non-red. Such a criterion must be part of the meaning of "red". The image and its phenomenological qualities are inner and private in the psychological sense. We call the image and its phenomenological qualities for private meaning. Since we, and other self-aware animals, can be conscious of redness and its distinguishing criterion before we learn any word for redness, private meaning is ontologically prior to language.

(2) The association of the word "red" with its private meaning. Such an association is, in the present example, brought about by a learning process based on an ostensive definition. It is part of the definition of the recursive semantic function F introduced in Analysis 1.6. For some words, there are other ways than ostensive definitions to learn their meaning, for example explicit or contextual definitions.

(3) The use of the word "red". The association of a word with its private meaning determines the use of the word. The association gives the rule for the use of "red". In the examples in Section 1, we found two types of uses. In a first-order use, the criterion of redness is applied but not referred to. First-order uses occur in the labelling of objects as red or non-red. In a second-order use, there is an implicit reference to the existence of the phenomenological qualities of redness. Aesthetic statements on redness are examples of second-order uses of "red". The use of a word is the public reflection of the private meaning. We may call use a public representation of meaning.

Private meaning, together with the rule for the association of a word with its private meaning, determines the use. Use does not determine private meaning. As shown in Analysis 1.25, one and the same use of "red" is compatible with more than one algorithm for redness. In particular, it is compatible with more than one
criterion of redness (which is the private meaning). Therefore private meaning is meaning. Meaning is prior to language and language is an instrument for the expression of meaning, as claimed in theses (2-3) and (2-4). Meaning is not primarily use. The use of a word or expression is determined by a rule. The rule connects the word or expression with a possible component of the mind or a possible mode of the mind. Therefore the rule connects the word or expression with something inner in a way which Wittgenstein excludes by the private language argument. For Wittgenstein, the rule connects the word or expression with something outward, namely a set of language games.

Language is used to express meanings. It is not used to express use in the language. Therefore meaning is not use in the language. Language is normally used to communicate what is on a person's mind. Therefore meaning is normally identical with something that is on a person's mind.

Sometimes we communicate most efficiently what is on our minds by not using the words as they are normally used. Thus Kafka's dictum "If you don't kill me, then you are a murderer!" cited in Remark 2.2 expresses in its paradoxical and metaphorical way much more effectively what was on Kafka's mind than the pedantic interpretation of it in the same remark. The paradoxical character of Kafka's statement is a consequence of the fact that he does not use some words as they are normally used, while the interpretation uses the words in the normal way.

2.4 IDEA. When I first some years ago heard that chimpanzees (or, more precisely, bonobos) can learn to speak (a fragment of) a human language, I immediately connected it with the well-known fact that chimpanzees are self-aware and self-conscious. This gave the first hypothesis:

(4-1) Self-awareness is a necessary condition for meaning.

The next idea came immediately after:
(4-2)  

*Self-awareness is a sufficient condition for meaning.*

Here are a couple of reasons for believing in Thesis (4-1). No animal is known who can understand an essential fragment of a language, that is, phrases and sentences beyond simple commands, without having self-awareness. To mean something, a being must be able to intend to communicate the meaning to another being which is not possible without self-awareness. For Thesis (4-2), we consider perception. If a self-conscious individual sees a scene and is aware of his seeing the scene, then the scene is on his mind. Then it is something he, in principle, can want to communicate to another individual, that is, a meaning. A similar case can be made for thoughts, fantasies, memories, ideas, and intentions. This leads to the following thesis.

(4-3)  

*The meaning of a sentence is what the speaker wants to communicate by the sentence. Normally this coincides with something in the speaker's consciousness.*

Thus meanings exist in all self-aware beings: human beings, chimpanzees, bonobos, orangutangs, gorillas, dolphins, and elephants. Language arises much later in the evolution, as an instrument for the communication of content of consciousness. It took a considerable evolutionary pressure to develop an efficient instrument for the communication of the content of mind: language. Though some earlier human species, like the Homo Neanderthalis, no doubt were able to speak, really complex languages, as we know them today, seem to have arisen only with Homo Sapiens Sapiens.

As an example, consider two chimpanzees A and B standing turned towards each other. A, but not B, sees a flock of lions approach them behind B's back. By gestures, mimic and sounds, A makes B turn around so that he, B, also sees the lions and the danger. The lions and the danger are on A's mind. By making B perceive the same as he does, he takes care that the lions and the danger also come to B's mind and attention. This is his non-linguistic way of communicating
his content of mind to B. When it is done, B understands what A means, namely that a flock of lions is approaching. If A and B instead had been two human beings, A could have communicated the same meaning and content of mind to B linguistically by saying for instance: "Watch out, brother B! A flock of lions is approaching us behind your back." A language enhances enormously our possibilities of communicating the contents of our minds, that is meanings, to other people.

The primary purpose of language is for a speaker (or writer) to communicate to other human beings what is on his mind. Language can be used for a number of other purposes also. Thus a person may write down notes to support his own memory without intending any other person to read the notes, for instance in order to remember what to buy in the supermarket. What the person does is to take a piece of information which is on his mind at a time \( t_0 \) and communicate it to himself at a later time \( t_1 \). A person may even write down his thoughts with the sole purpose of clarifying the thoughts, without intending either himself or anybody else ever to read them. It is a well-known experience that it is often useful to follow such a procedure. The kind of thinking for which this works is thinking which consists in silently speaking to oneself. One reason why the procedure works and is useful is that the explicit formulation of the thoughts in writing forces an intellectual (logical) discipline upon us which is not always present when we only think and silently speak to ourselves. Another reason is that the recording of the thoughts in writing supports the memory which is instrumental in checking the validity of the maybe complex set of ideas and in the potential development of the ideas.

2.5 OBJECTION. When we see a red object, it is the object itself we see and not the image of it. Then also the criterion of redness must be connected with the object itself and not with the image of it.
REPLY:

When we see a red object, a mechanism which involves the eyes and parts of the brain is active. The perception creates a state in the brain which is immediately accessible to our consciousness. The consciousness has no direct access to the object. Therefore the basic criterion of redness can only be in terms of the inner state. An automatic projection operation then projects the redness out on the external object. The existence of such a projection operation can be proven by a simple experiment. It was communicated to me and the rest of the class by our biology teacher in secondary school, Miss Rigmor Holt. Take a peg! Hold one end of it firmly in the hand! Move the other end over an uneven surface! When we thus use the peg as a tool, we feel an unevenness which we automatically locate to that end of the peg which is in contact with the surface. We certainly do not feel the unevenness directly because there are no nerves in the peg. What we do feel directly are the vibrations in that end of the peg which we hold in the hand. They are the basic criterion of unevenness. By an automatic projection, we then locate and feel the unevenness of the surface at the other end of the peg. (If we hold the peg only with a loose grip, the automatic projection disappears and we feel only the vibrations in that end of the peg which we hold in the hand. Now the peg is no longer used as a tool and as a prolongation of the arm and hand.) The case with the perception of the red object is similar. We are directly aware of the state in the brain. The basic criterion of redness must be formulated in terms of it. But by the automatic Miss Holt projection function, we assign redness to the external object, and we do, indeed, see the object as red and not the image as red. In the same way, many other things we have in mind can be and are projected out in the external world.

This projection operation presupposes a belief in the existence of the external world. This is an assumption which nearly every human being adopts. We do it quite early in life. It is part of our becoming self-conscious. Being self-conscious
involves being able to imagine ourselves as seen from the outside as objects in a world larger than ourselves. We cannot prove the existence of an eternal world. Why do we accept this hypothesis if it cannot be proven? Because it is part of self-consciousness, and being conscious is an enormous advantage from a social, and therefore evolutionary, point of view. For instance, as we have seen, without self-consciousness there can be no language. The presence of a language in a group of people increases enormously the efficiency of the group as is well known.

2.6 OBJECTION. The identification of meaning with something that is on an individual's mind makes meaning psychologically internal. Meaning can be communicated, while something inner cannot. Therefore this theory of meaning must be false and the private language argument be right after all.

REPLY:

Our criterion of redness is the phenomenological character of the colour. It is something inner. The criterion of redness must be part of the meaning of the word "red" which therefore contains a reference to something psychologically internal. The objection can therefore not be correct. The problem here is to show how meaning being internal is compatible with the communicability of meaning. First consider the predicate "red". Its meaning is determined by the algorithm used to distinguish red objects from non-red objects and, in particular, by the criterion of redness in the algorithm. Let us say that two algorithms are equivalent if they always give the same output for the same input. Suppose I and another speaker U both have learned the meaning of "red". Then our algorithms and criteria of redness need not be identical. If so, the private meanings we assign to "red" are not identical. However, our algorithms and criteria of redness must be equivalent. If they are equivalent, we use "red" in the same way. Thus two speakers can use a word or a phrase in the same way and still assign differ-
ent (private) meanings to it. One individual V can teach another individual U the use of the word "red"; but he cannot teach U the meaning of "red". U himself supplies an algorithm and a criterion for redness which fit the use. Then and only then does U assign a meaning to "red". The meaning is not uniquely determined by the use; but for U, the meaning determines the use.

Suppose I watch an object G and see that it is red. I inform U of this by the sentence

(6-1) Object G is red.

We assume that U knows which object is named by "G". By the theory of meaning in (2-2), the meaning of the sentence (6-1) is what is on my mind. By the projection function, what is on my mind is an external state of affairs, a fact. The fact can be formulated as follows:

(6-2) If an individual applies his visual algorithm for redness to object G and his algorithm is equivalent with mine, then the outcome will be affirmative.

A fact is determined by its verification algorithm. Facts are not in general what true statements state. Because of the effect of the projection function, the formulation (6-2) gives a public representation of the meaning of the declarative sentence (6-1) rather than the meaning itself. We see that the private meaning of "red" does not occur in the public meaning representation of "Object G is red" because (6-2) only demands equivalence of the algorithms, not identity. Speaker and hearer communicate by public representations of meaning. Each understands the message by supplying his own private meaning to the predicate "red". When we generalise this insight, we get the following results:

(6-3) The meanings of words and phrases are primarily private.

(6-4) The meaning of a word or phrase occurring in a sentence does not enter into the public representation of the meaning of the sentence -
be it declarative, interrogative, commanding or whatever. What does enter into the public representation of the meaning of the sentence is the public representation of the meaning of the word or phrase. It only demands equivalence, not necessarily identity, of the algorithms associated with the word or phrase, and therefore it only demands the same use, not necessarily the same meaning, of the expression. Speaker and hearer each supplies his own meaning to the expression. This makes communication by means of sentences possible.

(6-5) We communicate with each other only by sentences and not by single words and phrases.

(6-6) The meaning of a sentence p is what is communicated by p, and in the normal and primary case, this is something that is on the speaker's mind.

(6-7) Contents of mind (like propositions, beliefs, theories, dreams, imaginations, fantasies, intentions, queries) are the primary units of meaning. They are the meanings of whole sentences. The meanings of words and expressions are components of the primary meaning units. They are the results of analyses of some primary meaning units. A language community agrees on the use of words and expressions to designate such meaning components. A user associates each word or expression, he knows the use of, with a private meaning component. The words and expressions thus become tools by which he can express the contents of his mind. This is a kind of semantic holism where the meaning of a sentence is more fundamental than the meaning of the words and expressions occurring in it. Once meaning components have been identified and connected with words or expressions in a language, the latter can be used as tools to express new contents of mind (meanings) in new sentences. Even for
these sentences, the meaning of the sentence comes first to the
speaker's mind and next comes the search for suitable words (in-
struments) to express the meaning in a sentence. A hearer focuses
first on the meaning of words and phrases in the sentence. From the-
se meaning components together with pragmatic circumstances
around the communication, he tries to figure out the meaning of the
sentence, and normally this coincides with figuring out how the
speaker thinks and what is on his mind.

(6-8) Meaning components, the meanings attached to words and phrases,
are the results of a top-down analysis of sentence meaning. The
meanings of multi-sentence systems like accounts, stories, and theo-
ries are mainly built bottom-up from sentence meaning. Some sen-
tences get part of their meanings from neighbour sentences (the con-
text).

I now apply the theory of meaning developed in the present section to some
questions in the philosophy of language.

2.7 Thinking and language. What is the meaning of words like "thinking" and
"thought"? According to the private language argument, they cannot refer to
something inner. The criterion for the presence of a thought and thinking must
be something outward. We might imagine that the outward phenomenon is a
tendency to some type of movement. A trouble with this idea is that the move-
ments are often not sufficiently specific to identify a thought uniquely. Suppose
I think that I play football on Wembley Stadium in London. Then there might be
a tendency to movements in my legs and body associated with running and kick-
ing from which it might be possible to infer that I think about playing football;
but it should be impossible from the movements to infer that I think about the
playing taking place on Wembley. An alternative outward criterion of thinking
which avoids this objection could instead be (silent) talking to oneself. Several followers of Wittgenstein and the Oxford philosophers have identified thinking with silent talking to oneself.

2.8 REMARK. An empirically based objection to this idea is that some animals without language, like wild chimpanzees, clearly can think and solve problems. The thesis in §2.7 is a consequence of the private language argument. In Section 1, the private language argument was rejected. Then we need not accept its consequences either. A consequence of the thesis in §2.7 is that all thoughts can be expressed in one or more sentences. Consider a thought expressed in a sentence \( p \). Clearly the thought should be identified with the meaning of \( p \) rather than with \( p \) itself because the thought can be expressed by several different sentences if only they have the same meaning. But, as we have seen, meanings can exist before language. Therefore also thoughts can exist before language.

Thinking can be identified with problem solving. It is fairly easy to give examples of thinking which is not linguistic. Simple mechanical problems can be solved without being formulated in a language. Such problems are constantly solved by chimpanzees, both in captivity and in the wild. All that is needed is an ability to perceive the relevant mechanical system and to imagine the change in and manipulation of the system which solves the problem. Similarly some social problems can be solved without ever being formulated in any language. Via the self-consciousness — which implies an awareness that even other individuals of our species are self-conscious — and a hermeneutic method, many a psychological and social problem in a group can be solved without language being involved. Even in human beings with full linguistic competence, much thinking is non-linguistic. This is the case with thinking based on silent knowledge. Much of an artisan's competence is of this silent sort. Therefore also much of his thinking during his work is non-linguistic. If our thinking were exclusively in terms of words, figures in textbooks should not be needed. They should not the least
help the reader in the understanding of the text. The term "image" here includes all kinds of impressions associated with the senses, for instance visual, auditory, and tactile images. Much of a musician's or composer's thinking is in terms of auditory images. Such thinking might be called thinking in terms of images. Another form of non-linguistic thinking could be called operational thinking.

Sometimes we remember a phenomenon X by remembering the bodily movements associated with X rather than X itself. I remember for instance the code to the lock of the main entrance in the house where I live by remembering the pattern of motions used when I press the buttons at the entrance rather than by remembering the ciphers themselves. If somebody asks me for the code, I first have to imagine the motions in the mind to reconstruct the number. A pianist often, at least partly, can remember a complicated piece of music only by redoing concretely or in the mind the movements of fingers and feet during the playing of the piece. This can be generalised to other kinds of operations we make, for instance operations of and in the mind. Corresponding to the operational way of remembering, there is an operational way of thinking. Again musicians, composers, and artisans problem solving can be adduced as examples.

On the other hand, it is clear that a large part of our thinking does consist in silently speaking to ourselves. And the possibility of thoughts formulated in a language enormously increases our capacity for thinking. Clothing our thoughts in words makes it possible for us to think abstractly. Expressing thoughts in sentences makes it possible for us to construct long chains of reasoning and construct theories and other comprehensive systems of thoughts far beyond anything which is possible in wordless thinking. Einstein is reported to have said "My pen is smarter than I am" meaning that Einstein with a pen and paper is more than twice as smart as Einstein without these utilities. Similarly, Wittgenstein could have said (but didn't) "My language is smarter than I am" meaning that Wittgenstein with a language is more than twice as smart as Wittgenstein without it. But there is a large and unbridgeable gap between this and the con-
clusion that all thinking is linguistic. Most of our intellectual activity is a combination of operational thinking, thinking in terms of images, and thinking in terms of words.

2.9 **Universal grammar.** Chomsky has tried to explain the amazing ease with which children learn their native tongue by postulating the existence of a *Universal Grammar* containing features which are common to all natural languages. The knowledge of the universal grammar is inborn in human beings. This knowledge is claimed to be domain-specific by being effective for the peculiar matter of language learning.

2.10 REMARK. Chomsky's ideas are not compatible with the theory of meaning stated above. Meaning is conscious content of the mind. Meanings precede sentences. Sentences are instruments for the expression of meanings. Whatever is common to all grammars should come from the meanings alone. But meaning, being the content of mind, is in itself non-linguistic. Therefore, if anything is universal in the grammars of natural languages, it cannot be language specific.

Chomsky nevertheless has a partly correct intuition. We first consider the simplest languages which are sufficient for the expression of all declarative sentences about the world, the languages of predicate logic. Their non-logical symbols fall in three categories: (1) constants, (2) predicates, and (3) function symbols. The corresponding ontology consists of: (1) individuals (named by constants), (2) properties and relations (named by predicates), and (3) functional relations (named by function symbols). These are the ontological categories which are used in the perception of the world. We see the world as a collection of individuals. We characterise the individuals by the properties they have and the relations which hold between them. We understand the working of systems by the functional relations obtaining between the parts. This is how our minds work at the most basic level. It gives rise to a non-linguistic syntax. For in-
stance, we perceive that an individual $a$ has a property $P$ (in predicate logical languages often expressed as $P(a)$). Therefore the subject-predicate structure of a grammar is, indeed, inborn in human beings. But it is part of our innate cognitive faculty and is not language-specific. The same subject-predicate structure is innate in other self-aware, languageless beings who see the world essentially as we do, like for instance chimpanzees. What is needed for the child is to learn exactly how the subject-predicate structure is represented and expressed in the natural language he or she is learning. In English, to take an example, the grammatical subject cannot be omitted. Speakers of English say for instance:

I am; you are; he, she, it is human; etc.

In Spanish, the grammatical subject is mostly omitted and is implicit in the verb-form or the context:

Soy; eras; es humano; etc.

A predicate logical language also contains logical symbols like variables, quantifiers, and connectives. They also represent simple modes of functioning of the conscious mind. Thus 'A or B' is an exhaustive and exclusive list of alternative solutions to a problem which an individual with some imagination and analytical ability can produce. 'If A then B' represents a functional connection between an input event described by A and an output event described by B which an individual with a sufficient understanding of the functional connection can intuit. A predicate logical language gets closer to natural languages in expressive power by being enriched with further expressive options like tenses, interrogatives, modalities, and expressions for propositional attitudes. They also represent pre-linguistic function modes of the mind. Thus tenses express the mind's ordering of events and actions in time. Propositional attitudes like 'I know that A' and 'he believes that B' are implicit in the self-consciousness of the mind, including the awareness that even other human beings are self-aware. In general, everything
which can be the content of a conscious mind can also be a meaning component of a sentence.

To sum up: There is, indeed, a kind of innate Universal Grammar. It pertains, however, to our general cognitive faculty and is not, in contrast to Chomsky's claim, language-specific. Human beings, as well as other primates, have a certain way to perceive the world and themselves. This way is inherited and genetically determined. It is not culturally conditioned. This mode of perception determines a framework for meanings and for the (deep) syntactic structures of our languages. Our language is a natural outgrowth of the way we perceive the world and ourselves. The fact that meanings and cognition-based syntactic structures are present in our minds before we start to learn our first language partly explains the ease with which young children learn the language. The other part of the explanation is that children and young persons have an innate appetite for and ability to learn and remember a large number of words and grammatical combinations and connect them with meanings and situations. It is not clear whether that ability is language-specific because young children also have an innate appetite and ability for adapting to their non-linguistic physical and social environment.

We see that contents of minds (like perceptions, images, queries, emotions, and operations) constitute meanings. Language (words, expressions, and sentences) is developed primarily to communicate to others the contents of our minds. Therefore, for informal, natural languages, meanings precede language. Why should we develop instruments for expressing something if we had nothing to communicate? For formal, artificial languages, the order is the opposite: language precedes meanings. First the language is given and then we give it meaning by interpreting it in for instance a set theoretical model or in game theoretical semantics. Formal languages are sometimes useful models of natural languages, in linguistics and philosophy of language; but the present observation
shows that the analogy must be used with great care. In essential respects, the
two types of languages are different.

2.11 **Language of thought.** A common idea among psychologists and philoso-
phers is that the brain (and mind) is a Turing machine. Its essential cognitive
activity, including thinking, must therefore consist in computations. This has led
some philosophers, for instance Harman and Fodor, to the hypothesis of a lan-
guage of thought by the following sequence of reasoning.

Computations presuppose representations. Thus computations of a Turing ma-
chine or a computer are often defined over numerals that represent numbers, or
over another formal language. The suggestion is then that the sort of computa-
tions in the brain which are thinking requires the kind of representations which
are normally used for thoughts, namely sentences. The language of thought is a
formal language encoded in the brains of human beings (and possibly other in-
telligent creatures) as a vehicle for their thoughts. The claim is that thinking
consists in performing computations on sentences belonging to this language of
thought (which is innate). Inferences in this language are unconscious causal
processes.

2.12 REMARK. (I) The language of thought hypothesis is not compatible with
the picture of language exposed in the present section. This picture explicitly
presupposes that there is thinking which is not represented in any language. An
analysis reveals what is wrong in the chain of reasoning that leads to the lan-
guage of thought hypothesis. A computation is a physical process. It operates on
physical states. Thus a computation in a computer operates on electronic marks.
To interpret the physical process as a computation, the physical states must be
taken as representations of something else. Thinking consists in computations on
physical states in the brain. These brain states can be, but need not be, interpret-
ed as representing sentences. In the case of thinking about a mechanical prob-
lem, for instance, they can be interpreted as representing images of states in the mechanical system under consideration. Much human thinking is in terms of images which do not represent sentences. Images are analogue representations in the mind of states, processes, and events in reality. Thinking in terms of images is a kind of computation on these images. No language in the proper sense of the word is needed. Therefore the language of thought hypothesis is essentially false. Some thinking is non-linguistic and in terms of images. Some thinking is linguistic and consists of silently speaking to oneself. Only in the latter case is there a language of thought, the natural language in terms of which we think. It is the only kind of language of thought there is.

(II) One part of the language of thought hypothesis states that the language of thought obeys a logic similar to predicate logic. The brain is so constructed, it is claimed, that if it is in a state which represents the premises (represented in the language of thought), then it is sometimes caused to enter a state which represents the conclusion. Thus, in the case of the language of thought, inference is a causal process. This causal logic is a consequence of the way the brain is constructed and therefore is innate. However, empirical investigations show that there is no innate logic in human beings. We must have a logic in order to solve problems and therefore all normal grown-up humans have a logic. But strong empirical evidence suggests that we acquire our logical competence in the same way in which we get other vital skills: by adapting to the environment in which we grow up and live. The Russian psychologist A. R. Luria made in the 1930s investigations of the reasoning ability of peasants in Kirghizia and Uzbekistan. His purpose was to clarify the role of literacy in the logical competence of people. Here is an interview with an illiterate Uzbek villager. Q is the interviewer and P the peasant.

Q: "In the far north where there is snow, all bears are white. Novaya Zemla is in the far north, and there is always snow there. What colours are the bears there?"
P: "There are different sorts of bears. [The syllogism was repeated.] I do not know. I have seen a black bear; I have never seen any others …. Each locality has its own animals: if it is white, they will be white; if it is yellow, they will be yellow."

Q: "But what kind of bears are there in Novaya Zemla?"

P: "We always speak of what we see; we do not talk about what we have not seen."

Q: "But what do my words imply?" [The syllogism was repeated.]

P: "Well, it is like this: our tsar is not like yours, and yours is not like ours. Your words can be answered only by someone who was there, and if a person was not there, he cannot say anything on the basis of your words."

Q: "But on the basis of my words, 'in the north, where the bears are white', can you gather what kind of bears there are in Novaya Zemla?"

P: "If a man was sixty or eighty and had seen a white bear and had told about it, he could be believed, but I have never seen one and hence I cannot say anything!" [At this point, a young Uzbek volunteered: "From your words, it means that the bears there are white."]

Q: "Who of you is right?"

P: "What the cock knows how to do, he does. What I know, I say, and nothing beyond that!"

The peasant shows great intellectual integrity. He expresses the consequences of a purely empirical methodology. Suppose a modern, western researcher has detected a certain frequency of a medical phenomenon in a large group of white, Caucasian men. If asked "How is it with white Caucasian women?", he might answer, in the same vein as the Uzbek peasant, "I don't know; we have not examined it." To the question "How is the frequency among Asians and Africans?", the researcher's answer should be the same "I don't know; we have not examined it." (Compare with the peasant's declaration: "We always speak of what we see; we do not talk about what we have not seen.") Everybody, includ-
ing an elderly Uzbek peasant in the 1930s, needs a logic. Otherwise thinking and problem solving become impossible. Peasant P's answers to the interviewer Q show that his logic is empirical, and not a post-Aristotelian theoretical logic. Had the peasant had an inborn logic of thought akin to the usual predicate logic, he should immediately have given the answer which the post-Aristotelian interviewer expected. An empirical logic can only be learned and can therefore not be innate. This does not mean that the mind of a human baby is a tabula rasa. It is not. Observations of small children show quite clearly that they for instance have an innate idea of rules. A rule is essentially the same as a recursive function. The learning process of the child consists to a large extent in learning or discovering concrete rules and thus filling the idea of a rule with content. This is compatible with acquiring logical rules based on experience, as the Uzbek peasant in Luria's interview did, or logical rules based theoretical considerations, as most educated people do (and especially those who have taken courses in logic). The rules of theoretical predicate logic are not innate. (Similarly, the same innate idea of a rule makes it possible for us to develop an understanding of causal connections.)

(III) I repeat the premises on which the language of thought hypothesis is based: The brain (and mind) is a Turing machine. Therefore thinking consists in computations. Computations presuppose representations. A computation consists in the manipulation of the representations. (The language of thought hypothesis is then that these representations are sentences which are innate in our brains and are logically, ontologically, and temporally more fundamental that the sentences of the language we eventually learn to speak.) The premises appear to me to be true. Then what are in my philosophy of language the representations the manipulation of which thinking consists in? The picture of language given in Remark 2.10 gives the answer. At the most basic level, they are the representations in the brain of the individuals, properties, relations, and functions by which we perceive and conceive of the world and ourselves. These representations are
non-linguistic, and the manipulation of them is all we need for non-linguistic thinking. At a somewhat higher level, the set of representations in the mind is enhanced by the words and expressions of a natural language. Combining them into sentences, our thinking is developed to include also linguistic thinking. The natural language is our language of thought, and our only language of thought. The basic representations of individuals, properties, relations, and functions in the brain are the basic building blocks of logic as can be seen from any textbook exposition of the set theoretical semantics of predicate logic. Rules which relate these basic representations to allow inferences can be developed on an empirical basis, as instantiated by the Uzbek peasant. This gives rise to an empirical logic. Such a logic can be, but need not be, linguistic. A more advanced step in the history of logic consists in the development of a theoretical logic, as done by Aristotle and refined by Frege and other logicians.

To sum up: The representations needed for computations in the brain, and hence for thinking, consist of the non-linguistic representations in the brain of individuals, properties, relations, and functions. They are sufficient for non-linguistic thinking. At a later and higher level, this set of representations can be extended by words, expressions, and sentences of natural languages. They make linguistic thinking possible. These natural languages, and sometimes constructed languages like note systems and mathematical symbolism, are the only languages of thought we ever use.

2.13 **Meaning and intention.** It is felt intuitively that there is a close relation between meaning, intention, and communication. In a classical article from 1957, Paul Grice has tried to analyse meaning in terms of intention and communication. His contention is that basically and primarily "a person M meant such-and-such by the sentence \( p \)" is equivalent with "M uttered \( p \) with the intention of inducing a belief to the effect that such-and-such by means of the recognition of this intention". Thus, according to Grice, successful communication, that is, the
attainment of adequate understanding in a hearer H, requires recognition from the hearer of the speaker's intention in the communication process. It seems clear that he identifies the meaning of $p$ with this intention.

2.14 REMARK. (I) No doubt meaning can be identified with intention in some contexts. Thus the meaning of an action is what is intended by it. But the meaning of a sentence cannot be identified with intention in the way Grice does. It is useful here to invoke the distinction between act and content. Meaning cannot be identified with an act of intending because the meaning of a sentence clearly is not an act; but neither can it be identified with the content of the intention. The content of the intention - that is, what is intended - is the induction of "a belief to the effect that such-and-such by means of the recognition of this intention". But this cannot be identified with the meaning of the sentence $p$ either because meaning precedes ontologically the sentence and any speech act in which it may occur.

(II) The meaning of $p$ is rather the content of $p$ which M tries to (intends to) communicate. It is normally identical with something which is on M's conscious mind and which he tries to transfer to the mind of another conscious individual in the situation considered in Grice's definition. Thus the meaning is the content which M wants to communicate and not the content of the intention itself. Grice's analysis of meaning can therefore not be valid.

As an example, consider the following situation. I am known as a notorious liar. Even U knows that I am a notorious liar, and I know that U considers me a completely unreliable person. It is true that $p$, but I want to make U believe that $\neg p$. I therefore tell U that $p$. As I expect, U reacts according to his opinion about me: "That man is a consistent liar. He should never inform me that $p$ if $p$ were true. Therefore $\neg p$ must be the case." What I intend by uttering $p$ is to induce the be-
lief that \( \neg p \) in U and not to induce the belief that \( p \); but the intention to induce the belief that \( \neg p \) is certainly not the meaning of the sentence \( p \).

(III) If Grice does not analyse the meaning of \( p \), what does he analyse? He analyses the meaning of an action, namely the speech act of uttering \( p \). One objection to Grice's thesis is that the meaning of \( p \) is such-and-such while the meaning of the speech act, in the normal case, is to induce the belief that such-and-such. They are not identical. In my analysis, the meaning of \( p \) is the content of \( p \) that such-and-such and not the intention to induce the belief that such-and-such. Another objection is that meaning logically and ontologically is more fundamental than any speech act and therefore should not be defined in terms of speech acts.

(IV) To motivate the qualification "normally" in Part (II) above, it is instructive to look at the case of lying. Suppose M utters \( p \) and is lying. Then M is not communicating what is on his mind. Rather he communicates what he pretends to be on his mind. Thus in this case, the meaning of \( p \) is not identical with what is on M's mind but what he pretends to be on his mind. We therefore get:

(14-1) *Primarily, when the speaker is sincere, the meaning of an uttered sentence is something which is on the speaker's mind and which he intends to communicate.*

(14-2) *Secondarily, when the speaker is not sincere (for instance in the case of lying or acting), the meaning of an uttered sentence is something which the speaker pretends to be on his mind and which he intends to communicate.*

Consequently, in my semantic theory,

(14-3) *The meaning of \( p \) is what is communicated by \( p \), and in the normal and primary case, this is something that is on the speaker's mind.*
2.15 Rule-following. Wittgenstein's view is that a word or expression is used correctly if it is used in accordance with a rule. The rule associates the expression with its meaning. This is in agreement with the ideas defended in the present essay. The difference lies in the definition of meaning. For Wittgenstein, meaning is use in the language. Therefore a Wittgensteinian rule associates a word or other expression with the set of language-games that is its meaning. In my philosophy of language, a rule associates a word or expression with a component of the content of mind or with a mode of the mind. The idea that the meaning of a word or expression is determined by a rule is not original with Wittgenstein. What is new in the later Wittgenstein is the claim that the rule associates the word or expression with a set of language-games (which in turn determines its use).

2.16 REMARK. The first objection to Wittgenstein's idea of a semantic rule is that it should put superhuman demands on the human memory. Each grown-up person knows several thousands of words and expressions. If each of them is associated with a whole set of situations together with information about how to use the expression in each situation, then the rule associated with the expression becomes quite complicated. Adding the complexities for all the thousands of words and expressions we know results in a body of knowledge beyond at least the capacity of my small head. Taking into consideration that some of us know several languages, the weight placed on the memory becomes unbearable. Thus I, to take a nearby example, speak three languages fluently and two more fairly well, and some people speak more than five languages.

Another objection is the following. When I look introspectively into my own mind to see how I associate a word like "red" with its meaning, the following picture emerges. I know the phenomenological qualities of different nuances of red. The simple semantic rule I follow consists in the association of the phenomenological qualities of redness with the word "red". Such a rule is easy to re-
member. The situations and language-games by which I once upon a time learned which meaning to associate with "red" are long forgotten. They are pedagogical means for learning the meaning of the word, not the meaning itself.

To sum up: If meaning had been use, most of us should be able to learn only a rather small fragment of a natural language. Fortunately, meaning is not use, except for a few special expressions, and we are able to master not only one but several languages of several thousands of words and expressions and different grammars. The meanings of words and expressions are determined by rules; but the rules are simple because they connect words with inner states and modes of the mind. They are not complicated rules which connect a word or expression with an external set of language-games.

2.17 Kripke on rule-following. Ostensive definitions are definitions taught by means of examples. Thus we learn the rule for the use of an expression by being confronted with a finite selection of examples of the use of the rule. Kripke claims that there is a major problem with this kind of definitions. Suppose I am being taught a rule (a recursive function) \( a \) by being informed about the first four values 2, 4, 6, 8. Thus I am informed that

\[
\begin{align*}
(17-1) & \quad a_0 = 2, \ a_1 = 4, \ a_2 = 6, \ a_3 = 8 \\

\end{align*}
\]

The trouble is that there are infinitely many ways of continuing the sequence. They give rise to infinitely many recursive functions, for instance corresponding to sequences with initial segments

\[
\begin{align*}
(17-2) & \quad 2, 4, 6, 8, 10, 12, 14, 16, \ldots \\
(17-3) & \quad 2, 4, 6, 8, 11, 15, 20, 26, \ldots \\
(17-4) & \quad 2, 4, 6, 8, 51, 49, 47, 45, \ldots \\

\end{align*}
\]

Each one corresponds to a rule and there is nothing in the information conveyed by the example (17-1) which indicates which of the three rules, or possibly a
fourth rule, is actually used in the example. Therefore a learner cannot know which rule the teacher tries to teach him.

The example used by Kripke is the following. He considers two two-place functions \textit{plus} and \textit{quus}. They are recursive functions defined by

(17-5) \[ m \text{ plus } n = m + n \]

and

(17-6) \[
\begin{align*}
  m \text{ quus } n &= m + n & \text{ if } m, n \leq 57 \\
  m \text{ quus } n &= 5 & \text{ if } m > 57 \text{ or } n > 57
\end{align*}
\]

where + in both definitions is determined by some standard addition algorithm. A person being taught addition by examples of addition of pairs of numbers which are all smaller than or equal to 57 cannot know whether the rule used and taught by the teacher is plus or quus (or possibly a third rule).

2.18 REMARK. Kripke's argument uses recursive functions. In (17-6), he uses definition of recursive functions by cases. It is therefore appropriate that we first define these concepts.

2.19 DEFINITION (\textit{Recursive functions}). The class of \textit{recursive functions} is the smallest class of functions closed under the rules (RF1)-(RF4).

(RF1) \textit{Basic functions: The zero functions}

\[ Z_n: \mathbb{N}^n \to \mathbb{N} \quad n \geq 0 \]

(19-1) \[ Z_n(x) = 0 \]

\textit{the successor function}

\[ S: \mathbb{N} \to \mathbb{N} \]

(19-2) \[ S(x) = x + 1 \]

\textit{the projection functions}
\( U_{n,i} : \mathbb{N}^{n} \rightarrow \mathbb{N} \)

\[(19-3) \quad U_{n,i}(x_1, \ldots, x_i, \ldots, x_n) = x_i \quad \text{for } n > 0 \text{ and } 1 \leq i \leq n \]

are recursive.

(RF2) Substitution: Let

\[
g : \mathbb{N}^{n} \rightarrow \mathbb{N} \quad \text{for } n > 0
\]

\[
h_i : \mathbb{N}^{m} \rightarrow \mathbb{N} \quad \text{for } m \geq 0 \text{ and } 1 \leq i \leq n
\]

be recursive. Then

\[
f : \mathbb{N}^{m} \rightarrow \mathbb{N}
\]

\[(19-4) \quad f(x) = g(h_1(x), \ldots, h_n(x))\]

is recursive where \( x = (x_1, \ldots, x_m) \).

(RF3) Primitive recursion: Let

\[
g : \mathbb{N}^{n} \rightarrow \mathbb{N}
\]

\[
h : \mathbb{N}^{n+2} \rightarrow \mathbb{N}
\]

be recursive. Then

\[
f : \mathbb{N}^{n+1} \rightarrow \mathbb{N}
\]

\[(19-5) \quad f(x, 0) = g(x)
\]

\[
f(x, y+1) = h(x, y, f(x, y))\]

is recursive.

(RF4) Minimisation: Let \( g : \mathbb{N}^{n+1} \rightarrow \mathbb{N} \) be recursive, and assume that

\[
\forall x \exists y \ g(x, y) = 0
\]

Then

\[
f : \mathbb{N}^{n} \rightarrow \mathbb{N}
\]

\[(19-6) \quad f(x) = \mu y \ (g(x, y) = 0)\]
is recursive where \( \mu y (\ldots y \ldots) \) denotes the smallest number \( y \in \mathbb{N} \) which satisfies the condition \( (\ldots y \ldots) \).

2.20 DEFINITION (Characteristic function). Let \( R \subseteq \mathbb{N}^n \) be an n-place relation (a set if \( n = 1 \)).

(I) The characteristic function \( K_R \) of \( R \) is the function

\[
K_R : \mathbb{N}^n \to \mathbb{N} \\
K_R(x) = 1 \quad \text{if } x \in R \\
K_R(x) = 0 \quad \text{if } x \notin R
\]

(II) \( R \) is recursive iff \( K_R \) is recursive.

2.21 LEMMA (Definition by cases). Let

\[
f, g_1, \ldots, g_k : \mathbb{N}^n \to \mathbb{N} \\
R_1, \ldots, R_k \subseteq \mathbb{N}^n
\]

such that for each \( x \) exactly one of \( R_1(x), \ldots, R_k(x) \) is true. Let \( f \) satisfy

\[
(21-1) \quad f(x) = g_1(x) \quad \text{if } R_1(x) \\
\ldots \ldots \\
\ldots \ldots \\
(21-1) \quad f(x) = g_k(x) \quad \text{if } R_k(x)
\]

If the \( g_i \) and \( R_j \) are all recursive, then \( f \) is recursive.

PROOF:

Since \( f \) has the explicit definition

\[
(21-2) \quad f(x) = g_1(x) \cdot K_{R_1}(x) + \ldots + g_k(x) \cdot K_{R_k}(x)
\]

and addition and multiplication are recursive, \( f \) is recursive.

2.22 REMARK (continued). (I) From Definition 2.19 and the identity (21-2), we see that every definition by cases can be replaced by an explicit definition which
does not use cases. Therefore the problem pointed out by Kripke can always be circumvented if we have the expedient of explicit definitions at our disposal. One of Wittgenstein's points is that in language learning, we do not always have the possibility of using explicit definitions; we are often confined to ostensive definitions. If we do not have the possibility of explicit definitions at hand, the problem pointed out by Kripke does arise in the framework of recursive functions in general because there is no upper bound to how many different cases there can occur in a definition by cases. I now show that this is of no relevance in the special case of language learning.

(II) We first consider the situation where linguistic competence is acquired in a teacher-learner relation. From identity (21-2), we see that a definition of a rule $f$ by cases must contain adequate information about all the different cases. If $f$ is defined by examples, the teacher is obliged to provide examples which adequately illustrate all the different cases $R_1(x), \ldots, R_k(x)$ and make it clear that they are all the cases. If the initial segment 2, 4, 6, 8 illustrates all the cases, it can only be an initial segment of the function $f(x) = 2(x+1)$. Any function with (17-3) or (17-4) as initial segment must be defined by a recursive procedure involving more than one case which are not all illustrated by 2, 4, 6, 8. Similarly, plus can be defined by only one case while quus needs at least two cases. In the case of quus, a definition by examples involving only numbers less than or equal to 57 does not contain adequate illustrations of all the cases. Therefore such examples alone can define only plus.

(III) Part (II) solves the Kripke problem when there is a teacher. Now consider the following situation. A person $P$ visits a foreign country whose language he does not know. $P$ tries to learn the language by observing how the native speakers use it. To do that, $P$ must try to learn the semantic and grammatical rules of the language from examples. Even in this case, Kripke's objection is not valid. The reason is that definitions by cases are used in a special way in natural lan-
guages. An expression like $f$ is assigned one meaning in situations which satisfy $R_1$ and another meaning in situations which satisfy $R_2$, etc. $P$ must learn to distinguish between the situations $R_1, \ldots, R_k$ and identify the meanings attached to $f$ in each type of situation. When $P$ has learned the meaning of $f$ in situations of type $R_1$, he can use $f$ in such situations. This does not lead him to use $f$ in the same way in situations of type $R_2$ because not all markers of situations of type $R_1$ are present in situations of type $R_2$. Examples which illustrate that natural languages work like this can be found in any dictionary. When $P$ has learned to attach the meaning $f(x) = g_1(x)$ to $f$ in situations $R_1$, then he has learned partially how to use $f$. He can then later add to his knowledge of the language by learning the meaning of $f, f(x) = g_2(x)$, in situations of type $R_2$, etc. There are at least two advantages about organising a natural language according to this pattern. Expressions like $f$ are in situations like $R_1$ associated with a natural pre-linguistic meaning which makes the association fairly easy to remember. Knowledge of the different uses of an expression, like $f$, can be learned step by step. This allows linguistic competence to grow organically. Moreover, a speaker rarely needs, and therefore rarely needs learn, the meaning of an expression in all cases of its use.

For grammatical rules, a similar situation with step-by-step learning occurs. For instance, it is at least partly conventional how verbs are conjugated. In this case, the problem of finding the right conjugation for a verb is eased by the fact that generally, for any natural language, there are a few main rules for the conjugation of verbs plus a number of exceptions from these rules. It must be so since otherwise the grammar becomes too complicated to learn even for native speakers. A student of the language must proceed by identifying the few main rules and, step by step, learn which verbs follow them and also the conjugation patterns of the exceptions to the main rules. For every step forward, he masters a larger portion of the language. The everything-or-nothing situation in Kripke's example with definition by cases does not arise in natural languages.
2.23 EXAMPLE. The *Longman Dictionary of Contemporary English* gives five different meanings for the noun *spark*: (1) a small bit of burning material thrown out by a fire or by the striking together of two hard objects; (2) a light-producing passage of electricity across a space; (3) a direct cause of an event regarded as a fire or explosion: *The spark that set off the war was the murdering of the prince*; (4) a very small but important bit, especially of a quality: *a spark of cleverness/politeness*; (5) *bright spark*: a clever or cheerful person.

The symbol 'spark' has the same role as the symbol 'f' in Lemma 2.19. The five cases considered are the cases $R_1$, ..., $R_k$. We see that we can learn to use correctly the expression 'spark' in any of the five cases without knowing the meaning and use of the noun in the other four cases. I knew, for instance, before I looked up the word in the dictionary, the first three senses of the word; but I did not know the senses (4) and (5). This ignorance of mine never prevented me from using 'spark' correctly in the first three senses.

2.24 REMARK. The analysis in Remark 2.22 is concerned only with Kripke's example based on definition by cases. It does not exclude that there might be examples of recursive functions which cannot be learned from any finite number of function values. What I show is that Kripke's example and argument as it is stated is not valid when it is applied to natural languages. Towards the end of Section 1 in "Remarks on Wittgenstein's Philosophy: Philosophical Method and Contradictions", I will return to the question about the relation between meaning and rule following.

2.25 Nominalism. Quite a number of philosophers are nominalists. They accept the existence of physical individuals like sentences, predicates, and numerals; but they deny the existence of abstract entities like propositions, properties, and numbers.
2.26 REMARK. I show that the philosophy of language outlined in the present essay is incompatible with nominalism. Rather it might be characterised as a kind of conceptualism.

(I) Propositions. Propositions are identified with proposed solutions to such problems which are quests for information. Problems certainly exist. The world is full of problems. Solutions also exist. Many problems have solutions and have found their solutions. We have the semantic square:

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Problem ——————————— Proposed solution
|                        (Proposition)
|                        |
| Question ——————————— Answer
|                        (Sentence)
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The proposition represents the meaning of the declarative sentence. We have seen that the meaning logically and ontologically precedes the sentence. Therefore problem solving cannot in general be reduced to linguistic activity. A problem is a quest for something which satisfies a set of conditions. These conditions define the problem. The ability to solve problems in this sense is a necessary condition for life and much more fundamental than language. Therefore propositions exist as proposed solutions to problems. By the Miss Holt projection function, the proposed solution is projected out on reality and identified with a fact. Problem solving cannot be reduced to a purely linguistic activity, a use of language. First there are, as shown, problems and problem solving which are nonlinguistic. Secondly, we have shown that meaning, and therefore also sometimes problem solving, precedes language formation. And thirdly, there are paradoxes in the philosophy of language, like the Liar paradox and Grelling's para-
dox, which can be solved in the semantics in § 2.6 but cannot be solved in Wittgenstein's nominalistic semantics in § 2.1.

(II) Properties. One argument against the existence of properties sometimes advanced is that it is not intelligible what a property is or could be. Let us consider the property red in an object $a$. Suppose a person P looks at $a$ and perceives that it is red. P applies his algorithm for colours including the criterion for redness, the presence of a specific phenomenological quality. This phenomenological quality is then, by the Miss Rigmor Holt projection function, projected out on the object $a$. Therefore we can define the property red as the disposition, in the present case in $a$, to elicit a certain phenomenological quality in the mind, in the present case in P's mind. This explains what the property red is and shows the existence of this property. Generalising the argument, many other properties can be shown to exist. This is a conceptualistic, non-nominalistic, and non-Platonic theory of properties.

(III) Numbers. The argument against the existence of numbers usually comes in two steps. First it is claimed that no numbers can be observed in the physical reality. Second, philosophical arguments are given against the existence of numbers somewhere else than in the physical reality, for instance in a Platonic world. Here is my argument for the existence of numbers. Examinations of the foundations of physics (for instance in my book Logical Physics: Quantum Reality Theory from 1996) show that operations are ontologically more fundamental than physical entities like fields and particles. Therefore operations exist in the physical reality. They are the fabric from which physical reality is built. Every operation can be performed by, and hence identified with, an algorithm. By Turing's thesis, every algorithm can be represented by a recursive function. Therefore operations are essentially the same as recursive functions. Natural numbers are identical with zero-place recursive functions. Therefore natural numbers exist in the physical reality, as zero-place operations. No Platonic reali-
ty is needed and relevant. This is just as it should be because it is in the physical world we need the numbers when we solve problems about physical reality. Other mathematical entities beyond natural numbers and recursive functions, like infinite sets, are conceptual constructions. They are the results of mental operations combined with suitable assumptions.

2.27 REMARK. The private language argument has, or ought to have, the following structure:

(27-1) By definition, a private language cannot be used for interpersonal communication. (Apparently Wittgenstein tries to show also that there is not and cannot be any private language; but he does not need this stronger conclusion for his purposes in philosophy of mind.)

(27-2) Since ordinary psychological language can be and is used successfully for interpersonal communication, it cannot be private.

(27-3) If psychological terms like "pain", "pleasure", and "sensation" had referred to something inner and private, they should belong to a private language.

(27-4) Therefore psychological terms do not refer to something inner and private.

There is some uncertainty about what Wittgenstein meant by the private language argument. In particular, what did he mean by "an inner and private state"? There are two reasonable interpretations: (1) A state in a person P is inner and private if and only if it belongs to the immaterial soul of P, in the sense of a Cartesian psycho-physical dualism. (2) A state in P is inner and private if and only if it is a state in P's central nervous system which is directly accessible to P himself via his self-awareness.
One interpretation of the private language argument could be that Wittgenstein only wanted to refute the existence of states of type (1) by showing that if states of type (1) existed, reference to an inner state would make the language private. (It is called the weak interpretation of the private language argument in Remark 1.38 above.) This weaker interpretation should make some of the statements and examples adduced by Wittgenstein in the private language argument intelligible.

The interpretation (the strong interpretation) of the private language argument actually used in Section 1 above is somewhat stronger, namely that Wittgenstein wanted to show the validity of Thesis (10-1) in § 1.10 which excludes reference to inner states in both of the senses (1) and (2) defined above. Some of the formulations in the paragraphs on the private language argument in Philosophische Untersuchungen support this interpretation. Even if his intention was to consider only inner, private states in the first sense, the interpretation he himself actually gives of the private language argument is that he has excluded reference to inner, private states in both of the senses (1) and (2). This can be seen from the fact that some of the consequences that he draws from the argument are valid only if he includes both senses of "inner and private state" in the argument. In particular, if Wittgenstein had been concerned only with states in the sense (1), he should not have been forced to identify meaning with use in the language in the way shown in the analysis in Remark 2.2 of the present section but should at least have discussed the alternative of identifying meaning with something in a speaker's brain and accessible by introspection. It appears that Wittgenstein failed, in the present context, to distinguish between the two senses (1) and (2) above of "inner and private state". Because the private language argument is the fundamental building block in Wittgenstein's later philosophy, for instance in the theory of meaning and in the therapeutic conception of philosophy, this failure turns out to be disastrous for his endeavours.

Wittgenstein is concerned with philosophy of mind in the private language argument. The philosophy of mind defended in the present essay is the following.
A normal human being has a brain. States, processes, and events in the brain can be studied from the outside (by for instance neurological and psychological methods and observation of behaviour); but they can also be studied from the inside. Via the self-awareness, the brain can observe some of its own states, processes, and events from the inside. Thus the mind satisfies the following characterisation.

(27-5) \[ \text{A person } P\text{'s mind} = P\text{'s direct awareness (both as act and content) of states, processes, and events in } P\text{'s own brain.} \]

This is mainly the Cartesian conception, though without any ontological psychophysical dualism. The analysis in Section 1 shows that the private language argument does not exclude this kind of philosophy of mind, and it shows that the private language argument does not exclude reference to inner and private states, processes and events in the sense (2).

2.28 Verification and Falsification. A part of the private language argument is Wittgenstein's claim that statements about what is in and on the mind must be verifiable. But if they are about something inner, they cannot be verifiable. And if they are not verifiable, we cannot know precisely what another person means by such statements.

First I am not convinced that statements about what is on a person's mind cannot be verified as a statement of an inner state, process or event in that person's brain. One day science may reach a state where it is possible to register such states, processes, and events in the brain and show how they appear to a self-conscious mind and make interpersonal comparisons between such appearances.

However, in everyday practical interaction between people, there can be no such verification, and hence not in language learning either. Therefore, in practice, I cannot verify that for instance the colour red appears the same to another person
as it does to me. I cannot verify that another person means the same by "red" as I do. My claim in Section 1 was that it really suffices with falsifiability of such statements. The theoretical reason why falsifiability suffices is that another mind in this context must be conceived of as a black box and for hypotheses about black boxes, falsifiability but not verifiability is possible; but still such hypotheses are meaningful. The practical reason why falsifiability suffices is that as long as I am not able to falsify the hypothesis that the other person understands the same by "red" as I do, then the social interaction functions between us as far as the communication by the word "red" concerns. If science one day reaches a level of development where it is possible to determine whether a red object appears with the same phenomenological qualities to two different persons, then the mind loses its status of being a black box and then it will also be possible to determine, and not only make hypotheses about, whether the two persons mean the same by "red", that is, whether they distinguish between red and non-red by the same phenomenological criterion. I personally believe that science will one day reach a level of development where it can be decided to what extent a red shade appears the same to the minds of two different people.

Language as it normally occurs presupposes a *community of minds*. We understand another person when we understand what is on the person’s mind. Another person’s mind is a black box. A hypothesis on the meaning of an expression uttered by another person is a hypothesis on the inner of the black box which is the other person’s mind. Such a hypothesis must be falsifiable to be a useful and reliable source of information on meaning. Wittgenstein’s demand that it must be verifiable is too strong and cannot be justified. The meaning of a word is not its use in the language, that is, the set of language games associated with it. The meaning of a word is based in the minds. For instance, the criterion we use to determine whether an object falls under the predicate "red" must be an essential part of the meaning of the word "red". But this criterion is for each individual based on the phenomenological appearance of red which is something inner in
the mind. The meaning in the language of the word "red" is what is common in
the phenomenological appearance of red in our minds. Given the hypothesis that
our minds are quite similar, we are able to communicate with each other and un-
derstand each other so well that the social interaction works acceptably well.
Other people’s minds are black boxes to us. But there is one mind, a person’s
own mind, which is not a black box to that person. Given this together with the
hypothesis that all people’s minds are quite similar, we can often quickly form
hypotheses about what is on another person’s mind and thus understand what he
means. The more different our minds are the more difficult is it for us to under-
stand each other immediately. But even such hindrances can be overcome. Thus,
in contradiction to Wittgenstein's well-known aphorism: If a lion could speak,
we should still be able to understand him. To a considerable extent, the "Le-
benswelt" of a lion and the "Lebenswelt" of a human being are similar. For in-
stance, the two species are both mammals. There are also considerable diffe-
rences between the "Lebenswelt" of lion and the "Lebenswelt" of man and hence
between their minds. Problems arising from such differences can be overcome
with the help of a zoologist specialised on lions. Lions do not speak; but bono-
bos are sufficiently self-conscious to learn a human language. If a bonobo
speaks, human beings have no problems understanding him or her. Our minds
are sufficiently similar to allow mutual understanding of our utterances, and
mostly experts on bonobos are not needed to facilitate the understanding.